basic integration

substitution method

The reverse operation to that which occurs in the chain rule…

$$y = f(u) \rightarrow where \rightarrow u = g(x)$$
$$\therefore \frac{dy}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

Integrating this…

$$y = \int \frac{df}{du} \cdot \frac{du}{dx}\, dx = \int \frac{df}{du}\, du = f(u) = f(g_{(x)})$$

trigonometric integrals

Use double angle formula and standard trig identities to simplify most trigonometric integrals then integrate using substitution method or integration by parts.

partial fractions

Complete partial fraction procedure as usual and then integrate.

For example:

$$I = \int \frac{1}{(x-1)(x+2)}$$

after working out partial fractions…

$$I = \int \left[\frac{\frac{1}{3}}{x-1} - \frac{\frac{1}{3}}{x+2} \right] dx$$
$$I = \frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x+2| + c$$

improper integrals

Integral is improper if $f(x)$ is not defined or continuous within the range $[a, b]$ or if the integral is infinite.

For example:

$\int_0^4 \frac{1}{x-1}\, dx$ is improper because $f(x)$ is not defined at $x = 1$

$\int_1^\infty \frac{1}{x+2}\, dx$ is improper as the integration range is infinite

integration by parts

Depends on the product rule, which when integrated gives…

$$\int u\, v'\, dx = uv - \int v\, u'\, dx$$

trigonometric substitution

Trig substitution is used for integrals involving;

$a^2 + u^2 \quad \sqrt{a^2 - u^2} \quad \sqrt{a^2 + u^2} \quad \sqrt{u^2 - a^2}$

For $\sqrt{a^2 - x^2}$ use $x = \text{a} \sin\theta \therefore dx = \cos\theta\, d\theta$
For $\sqrt{a^2 + x^2}$ use $x = a\tan\theta \therefore dx = \sec^2\theta\, d\theta$
For $\sqrt{x^2 - a^2}$ use $x = a\sec\theta \therefore dx = \sec\theta\tan\theta\, d\theta$

The substitutions are chosen as a means of eliminating the square root

definite integral

Includes bounds or limits

$$I = \int_a^b f(x)dx = F(b) - F(a)$$

Changing Limits

If you change the variable (i.e. u/trig substitution) then you must change the limits!

For Example:

$$I = \int_0^1 (x^2 + 1)^4\, dx$$

$$let\ u = x^2 + 1 \ \therefore du = 2x\, dx$$
$$when\ x = 0, u = 1\ and\ when\ x = 1, u = 2$$

$$\therefore I = \int_2^2 u^4 \frac{du}{2}$$

numerical integration

$$\text{h} = \frac{b-a}{2}$$

Trapezoidal Rule

$$\int_a^b f(x)dx \approx \frac{h}{2}[y_0 + 2y_1 + 2_{y2} \dots + 2y_{n-1} + y_n]$$

Simpson's Rule

$$\int_a^b f(x)dx \approx \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 4y_{n-1} + y_n)$$