expanding and factorising

binomial identities

F.O.I.L

$$(a+b)(c+d) = a(c+d) + b(c+d) = ac + ad + bc + bd$$

Perfect Squares

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

Difference of Two Squares

$$(a+b)(a-b) = a^2 - b^2$$

cubic identities

Expansion

$$(a+b)^3 = (a+b)(a+b)(a+b) = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a-b)^3 = (a-b)(a-b)(a-b) = a^3 - 3a^2b + 3ab^2 - b^3$$

Factorisation

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

inspection method

$$ax^2 + bx + c$$

$$a = 1$$

"We want 2 numbers that multiply to give c and add to give b"

Most useful when

two stage factorisation

$$ax^2 + bx + c$$

$$a \neq 1$$

$A \times B = ac$ → $(ax^2 + Ax)(Bx + c)$
$A + B = b$

~essentially inspection with an extra step~

completing the square

$$\boldsymbol{ax^2 + bx + c} = a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right] = a\left[\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a^2} + \frac{c}{a}\right]$$

(factor out a)

$$\frac{1}{2}\left(\frac{b}{a}\right) \xrightarrow{square} \left(\frac{b^2}{4a^2}\right)$$

$$= a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right] = a\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{\sqrt{b^2 - 4ac}}{2a}\right)^2\right] = a\left(x + \frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}\right)\left(x + \frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}\right)$$

(re-write in difference of two squares form) *(factorise using $a^2 - b^2 = (a+b)(a-b)$)*