fourier series

periodic functions

Fourier Series: a means of approximating a periodic function

Periodic Functions: repetitive

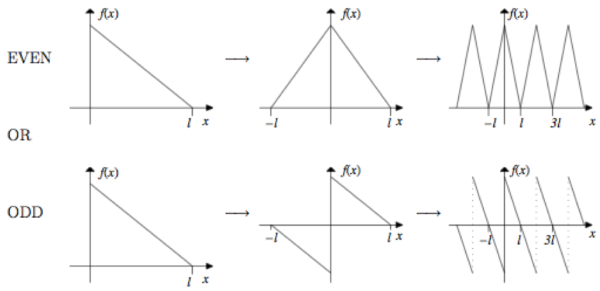
T : the period of the function

Why not just use a Taylor series?

1. *Polynomials are <u>not</u> periodic so they take a long time to converge*
2. *These functions often have discontinuities – which Taylor series can't handle.*

half-range series

~to approximate series over an interval (e.g.[0,1]) we use an odd or even extension of the function, i.e. a half-range series~



These are the even and odd periodic extensions of the function.

<u>Odd function: sin series</u>

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}$$

<u>Even function: cos series</u>

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$$

$$a_0 = \frac{2}{l}\int_0^l f(x)dx$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$$

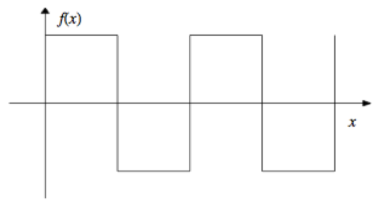
trigonometric series

~it makes sense to approximate a periodic function with a periodic function that we already know~

i. If f is an <u>even</u> function, then $f(-x) = f(x)$
   - Even function :. Use a cos series

ii. If f is an <u>odd</u> function, then $f(-x) = -f(x)$
   - Odd function :. Use a sin series

Example:

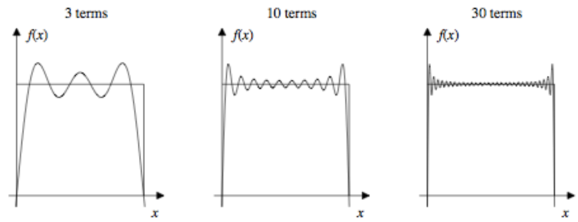


Odd :. Use sin series to approximate

:

$$\tilde{f}(x) \approx \frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \frac{1}{7}\sin 7x \ldots\right)$$

- The more terms = the more accurate



- The general formula to find the coefficients a_i and b_i ;

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left(a_n\int_{-\pi}^{\pi}\cos nx\,dx + b_n\int_{-\pi}^{\pi}\sin nx\,dx\right)$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos mx\;dx$$

$$b_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin mx\;dx$$

- If the period is T (rather than 2π then;

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\left(\frac{2\pi nx}{T}\right) + b_n\sin\left(\frac{2\pi nx}{T}\right)\right)$$

$$a_0 = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(x)dx$$

$$a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(x)\cos\frac{2\pi nx}{T}\,dx$$

$$b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(x)\sin\frac{2\pi nx}{T}\;dx$$